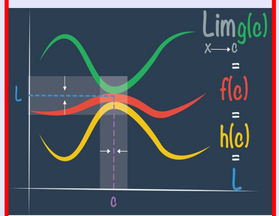
Calculus I

Lecture 23



Feb 19-8:47 AM

Class QZ 18 Open notes

Find abs. Max and abs. min of the Function

$f(x) = x^3 - 6x^2 + 30$ on $[-1, 5]$.

$f(x)$ is cont. on $[-1, 5]$

$f'(x) = 3x^2 - 12x$

$f'(x) = 0 \rightarrow 3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0 \quad , x = 4$

$f(-1) = (-1)^3 - 6(-1)^2 + 30$

$= -1 - 6 + 30 = \boxed{23}$

$f(5) = 5^3 - 6(5)^2 + 30$

$= 125 - 150 + 30 = \boxed{5}$

$f(0) = \boxed{30}$

$f(4) = 4^3 - 6(4)^2 + 30$

$= 64 - 6 \cdot 16 + 30$

$= 94 - 96 = \boxed{-2}$

$\boxed{\begin{array}{l} \text{Abs. max } 30 \text{ at } (0, 30) \\ \text{Abs. min } -2 \text{ at } (4, -2) \end{array}}$

Jul 24-7:18 AM

Substitution Rule with integration

$\int 3x^2 \cos x^3 \, dx$

$u = x^3$

$\frac{du}{dx} = 3x^2 \to du = 3x^2 dx$

$\int \cos u \, du = \sin u + C$

$= \boxed{\sin x^3 + C}$

Check:

$\frac{d}{dx}\left[\sin x^3 + C\right] = \cos x^3 \cdot 3x^2 + 0 = \boxed{3x^2 \cos x^3}$

Jul 24-8:18 AM

Find $\int x(x^2+9)^9 dx$

$u = x^2+9$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$= \int u^9 \frac{du}{2}$

$= \frac{1}{2}\int u^9 du$

$= \frac{1}{2} \cdot \frac{u^{10}}{10} + C = \frac{1}{20} u^{10} + C$

$= \boxed{\frac{1}{20}(x^2+9)^{10} + C}$

check:

$\frac{d}{dx}\left[\frac{1}{20}(x^2+9)^{10} + C\right] = \frac{1}{20} \cdot 10(x^2+9)^9 \cdot 2x + 0$

$= (x^2+9)^9 x$

$= x(x^2+9)^9$

Jul 24-8:23 AM

Find $\int x^2\sqrt{x^3+8}\,dx$

Hint:
$u = x^3+8$
$du = 3x^2\,dx$
$\frac{du}{3} = x^2\,dx$

$= \int \sqrt{u}\,\frac{du}{3}$

$= \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$= \frac{1}{3}\cdot\frac{u^{3/2}}{\frac{3}{2}} + C$

$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{9}u\sqrt{u} + C$

$= \boxed{\frac{2}{9}(x^3+8)\sqrt{x^3+8} + C}$

Jul 24-8:30 AM

Find $\int \frac{dx}{(1-6x)^4}$

Hint:
$u = 1-6x$
$du = -6\,dx$
$\frac{du}{-6} = dx$

$= \int \frac{1}{u^4}\,\frac{du}{-6}$

$= \frac{-1}{6}\int u^{-4}\,du$

$= \frac{-1}{6}\cdot\frac{u^{-4+1}}{-4+1} + C = \frac{-1}{6}\cdot\frac{u^{-3}}{-3} + C$

$= \frac{1}{18}\cdot\frac{1}{u^3} + C$

$= \frac{1}{18(1-6x)^3} + C$

Jul 24-8:37 AM

Evaluate $\displaystyle\int_0^{\sqrt{\pi}} 2x \cos x^2 \, dx$

$u = x^2$

$du = 2x\,dx$

$x = 0 \rightarrow u = 0^2 = 0$

$x = \sqrt{\pi} \rightarrow u = (\sqrt{\pi})^2 = \pi$

$$= \int_0^{\pi} \cos u \, du$$

$$= \sin u \Big|_0^{\pi} = \sin \pi - \sin 0$$

$$= 0 - 0 = \boxed{0}$$

Jul 24-8:42 AM

Evaluate $\displaystyle\int_0^{\pi} \sec^2 \frac{x}{4} \, dx$

$u = \dfrac{x}{4}$

$4u = x$

$4\,du = dx$

$x = 0 \rightarrow u = \frac{0}{4} = 0$

$x = \pi \rightarrow u = \frac{\pi}{4}$

$$= \int_0^{\pi/4} \sec^2 u \cdot 4\,du$$

$$= 4 \int_0^{\pi/4} \sec^2 u \, du$$

$$= 4 \tan u \Big|_0^{\pi/4} = 4\Big[\underbrace{\tan \tfrac{\pi}{4}}_{1} - \underbrace{\tan 0}_{0}\Big]$$

$$= 4(1-0) = \boxed{4}$$

Jul 24-8:46 AM

Evaluate $\displaystyle\int_0^{\pi/2} \cos x \,\sin(\sin x)\,dx$

Hint

$u = \sin x$

$du = \cos x\,dx$

$x = 0 \to u = \sin 0 = 0$

$x = \frac{\pi}{2} \to u = \sin\frac{\pi}{2} = 1$

$$= \int_0^1 \sin u\,du$$

$$= -\cos u \Big|_0^1 = -\left[\cos 1 - \cos 0\right]$$

$$= -\cos 1 + 1 = \boxed{1 - \cos 1}$$

Jul 24-8:52 AM

Find f_{ave} for $f(x) = \cos^4 x \sin x$ on $[0, \pi]$.

a b

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{ave} = \frac{1}{\pi - 0}\int_0^{\pi} \cos^4 x \,\sin x\,dx$$

$\cancel{u = \sin x}$

$\cancel{du = \cos x\,dx}$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$x = 0 \quad u = \cos 0 = 1$

$x = \pi \quad u = \cos\pi = -1$

$$= \frac{1}{\pi}\int_1^{-1} u^4(-du)$$

$$= \frac{-1}{\pi}\int_1^{-1} u^4\,du$$

$$= \frac{-1}{\pi}\cdot -\int_{-1}^{1} u^4\,du$$

$$\boxed{\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx}$$

$$= \frac{1}{\pi}\,\frac{u^5}{5}\Big|_{-1}^{1}$$

$$= \frac{1}{5\pi}\left[1^5 - (-1)^5\right] = \frac{1}{5\pi}\left[\underbrace{1-(-1)}_{2}\right] = \boxed{\frac{2}{5\pi}}$$

Jul 24-8:59 AM